# ~~Optional~~ Dimensional Analysis Extra Practice

**Dimensional analysis** converts one set of units to another just by multiplying by 1. In mathematics, you can always multiply by 1 and not change the value of a number. Further, if two numbers are equal to each other, then either number divided by the other is also equal to 1. So,

$$1 = \frac{4}{4} = \frac{\frac{1}{2}}{\frac{2}{4}} = \frac{100 \text{ centimeters}}{1 \text{ meter}} = \frac{1 \text{ pound}}{16 \text{ ounces}} = \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{60 \text{ minutes}}{1\text{hour}}.$$

If 1 mile = 5280 feet, then $\left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right) = 1$ and $\left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right) = 1$.

These "funny looking ones" are called **conversion factors**. We can use them to convert from feet to miles or vice versa. For instance, how many feet are there in 17.37 miles?

$$17.37 \text{ miles} \times \left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right) = 91713.6 \text{ feet}.$$

Notice that I multiplied by a conversion factor that made the "miles" in the numerator divide out with the "miles" in the denominator. If I had multiplied by the other conversion factor, the units wouldn't have divided out, and I would have ended up with

$$17.37 \text{ miles} \times \left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right) = 0.003290 \frac{(\text{miles})^2}{\text{feet}}.$$

The result is equivalent to 17.37 miles, but I wouldn't have answered the original question.

We treat the units of a measurement just the same as we treat the number part of a measurement. If the same unit appears in the numerator and the denominator, the common unit can be divided out; the unit of miles reduced out in the first example.

This method of converting units can be used in a string to convert more than one unit. For instance, the speed of a good fastball in professional baseball is about 90 miles per hour. To express this number in feet per second, do the following:

$$\left(\frac{90 \text{ miles}}{\text{hour}}\right) \times \left(\frac{1 \text{ hour}}{3600 \text{ seconds}}\right) \times \left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right) = \frac{132 \text{ feet}}{\text{second}}$$

Notice how the conversion factors were chosen so that the unwanted units divided out, and didn't multiply each other.

We can treat the units just as we would variables in an equation. If we were asked to convert 16.2 $\text{ft}^2$ (square feet) to $\text{in}^2$ (square inches), we could just square the appropriate conversion factor.

$$\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2 = \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) \times \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) = \left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right)$$

Now we have an appropriate conversion factor to convert square feet to square inches.

$$16.2 \text{ ft}^2 \times \left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right) = 2332.8 \text{ in}^2$$

This same method can be used to convert units of volume such as cubic yards ($\text{yd}^3$) and cubic centimeters ($\text{cm}^3$). Dimensional analysis can be used to convert any unit to any other appropriate unit.

Exercises:

*Use dimensional analysis to convert the following:*

1. 7 mi. to yards
2. 234 oz. to tons
3. 11.2 mg to grams
4. 1.35 km to centimeters
5. 9,800,000 mm (millimeters) to miles
6. 4.5 $ft^2$ to square yards
7. 435,000 $m^2$ to square kilometers
8. 8 $km^2$ to square feet
9. 0.0065 $km^3$ to cubic meters
10. 14.62 $in^3$ to cubic centimeters
11. 5,500 $cm^3$ to cubic yards
12. 3.5 mph (miles per hour) to feet per second
13. 185 yd. per min. to miles per hour
14. 153 ft/s (feet per second) to miles per hour
15. 248 mph to meters per second
16. 186,000 mph to kilometers per year
17. 7.50 $T/yd^2$ (tons per square yard) to pounds per square inch
18. 16 $ft/s^2$ to kilometers per hour squared

*Use dimensional analysis to solve the following:*

19. On a recent trip, Jan traveled 260 miles using 8 gallons of gas. How many miles per 1-gallon did she travel? How many yards per 1-ounce?
20. A chair lift at the Divide ski resort in Cold Springs, WY is 4806 feet long and takes 9 minutes. What is the average speed in miles per hour? How many feet per second does the lift travel?
21. A certain laser printer can print 12 pages per minute. Determine this printer's output in pages per day, and reams per month. (1 ream = 5000 pages)
22. An average human heart beats 60 times per minute. If an average person lives to the age of 75, how many times does the average heart beat in a lifetime?
23. Blood sugar levels are measured in milligrams of glucose per deciliter of blood volume. If a person's blood sugar level measured 128 mg/dL, how much is this in grams per liter?
24. You are buying carpet to cover a room that measures 38 ft by 40 ft. The carpet cost $18 per square yard. How much will the carpet cost?
25. A car travels 14 miles in 15 minutes. How fast is it going in miles per hour? in meters per second?

26. A cargo container is 50 ft long, 10 ft wide, and 8 ft tall. Find its volume in cubic yards and in cubic meters.

27. A swimming pool is 75 ft long and 54 ft wide. The pool is only partially filled with water, so the water surface is 6 inches below where it is supposed to be. How much water would it take, in cubic feet, to raise the water level by 6 inches?

28. A local zoning ordinance says that a house's "footprint" (area of its ground floor) cannot occupy more than $\frac{1}{4}$ of the lot it is built on. Suppose you own a $\frac{1}{3}$-acre lot, what is the maximum allowed footprint for your house in square feet? in square inches? (1 acre = 43560 $ft^2$)

29. Computer memory is measured in units of bytes, where one byte is enough memory to store one character (a letter in the alphabet or a number). How many typical pages of text can be stored on a 700-megabyte compact disc? Assume that one typical page of text contains 2000 characters. (1 megabyte = 1,000,000 bytes)

30. In April 1996, the Department of the Interior released a "spike flood" from the Glen Canyon Dam on the Colorado River. Its purpose was to restore the river and the habitants along its banks. The reservoir behind the dam contains about 1.2 trillion cubic feet of water. The release from the dam lasted a week at a rate of 25,800 cubic feet of water per second. About how much water was released during the 1-week flood? What fraction of the total water in the reservoir was released during the flood?

31. The largest single rough diamond ever found, the Cullinan diamond, weighed 3106 carats; it was used to cut the world's largest diamond gem, the Star of Africa that weighed 530.2 carats. How much does each diamond weight in milligrams? in pounds? How many grams of diamond was cut away to make the Star of Africa from the Cullinan diamond? (1 carat – 0.2 gram)

32. One watt is one joule per second. How many joules are in 1 kilowatt-hour (1 kilowatt × 1 hour)?

33. If one watt is one joule per second, how many joules does a 75-watt light bulb use in 1 week?

34. You are riding an exercise bicycle at a fitness center. The readout states that you are using 500 Calories per hour. Are you generating enough power to light a 100 watt bulb? (1 Calorie = 4184 joules)

35. Solar cells convert sunlight directly into electricity. At 100% efficiency, solar cells generate about 1000 watts of power per square meter of surface area when exposed to direct sunlight. If a house was equipped with a 2-square meter panel of solar cells that received the equivalent of 6 hours of direct sunlight per day, how many joules of energy can it produce each day? What average power, in watts, does the panel produce over a 24-hour period?

Selected Answers:

1. 12320 yd
3. 0.0112 g
5. 6.1 mi
7. 0.435 $km^2$
9. 6,500,000 $m^3$
11. 0.0072 $yd^3$
13. 6.31 mph
15. 111 m/s
17. 11.6 $lb/in^2$
19. 32.5 mpg; 447 yd/oz
21. 17280 pages/day; 103.4 reams/month
23. 1.28 g/L
25. 56 mph; 25 m/s
27. 2025 $ft^3$
29. 350,000 pages
31. Cullinan: 621,200 mg; 1.368 lb; Star: 106040 mg; 0.2336 lb
33. 45,360,000 j
35. 43,200,000 joules/day; 500 watts